ARTUR BERNAT*, WOJCIECH KACALAK**

TACKLING THE PROBLEMS
OF DETERMINATION AND NUMERICAL INTEGRATION
OF CONTENTS OF VECTOR GRADIENT FIELDS
IN POSSIBLE VISUAL INSPECTION
OF VERY TEXTURED SURFACES

Abstract: In this paper, some relevant information will be given, accordingly to the aimed problem: how to reliable acquire and numerically integrate noised contents of vector gradient field. In problems of visual inspection of very textured surfaces a multimage approach has been chosen, as a valid proposition for contactless examination of the surfaces characterised by both occurrence of complex topographic features on them, and complex reflectance-borne properties.

Key words: visual inspection, Photometric Stereo, surface contactless examination

1. COMPUTER VISION TECHNIQUES IN EXAMINATION OF THE DISTANT OR HARDLY ACCESSIBLE OBJECT SURFACES

In this paper some innovative method will be presented regarding both: aspects of efficient and reliable numerical integration methods of vector gradient fields in presence of noises, and reliable acquiring of consistent vector gradient fields.

It has been assumed, that the methodology here presented and minutely taken into considerations, should serve in mostly met in practice cases, where one very often deals with hard initial data acquisition conditions [2–5, 7].

This in turn means, that surface subdued to visual inspection, as an alternative approach to, for example, 3D tedious measurements, can be characterised by complex topographic-borne features, and moreover by complex reflectance borne features. Hence, the methodology here presented could be applied in cases, where the surfaces considered are either hardly accessible, in providing standard contact 3D measurements of their topography, or in cases where surfaces of distant objects are subdued to examinations.

* Mgr inż.
** Prof. dr hab. inż.  Fine Mechanics Division, Koszalin University of Technology.
2. SPECIFICS OF REFLECTANCE BORNE SURFACE FEATURES

Within methodology laboriously evolved by authors for visual inspections of the very textured surfaces, firstly it has been postulated that whole data processing framework of the data acquired should be based on multiimage approach to the problems. Thus, such a single-image approach, such as Shape-from-Shading (SFS) is rather out of scope of the works here related [9, 10, 12, 13, 20]. Instead of this, Photometric Stereo methods (PSM) are applied, jointly with some relevant and peculiarly distinct features embedded within them [2–7, 14, 15, 19, 21–23].

The reason for such proceedings here provided, within acquired methodology is the following. The whole data processing of the dataset of at least three 2D intensity images will be exclusively based on diffuse reflectance phenomena.

Hence, any shadowings, that means, occurrence of self-masking phenomena (of extremely low intensities resulted on 2D images within some locally spaced subareas, called also in Computer Vision literature the cast shadows) are to be excluded. Likewise, occurrences of self-shadowing phenomena (of low resulted intensities approaching nearly zero, called also in Computer Vision literature the
attached shadows), are also to be excluded. Analogously, any highlights, that means, occurrence of spatially acute and sharp specularities, and moreover occurrence of more spatially extended highlights, in reflections phenomena, specific for glossy surfaces, are to be excluded (Fig. 1).

Therefore, both extremely low resulted intensities and extremely high or highly elevated locally in their values resulted intensities, within dataset of several 2D images should be excluded, with use of double thresholding mechanism. Those intensities, which, in initial step of data classification and exclusion, are treated as invalid, are to be substituted with use of supporting data. That means, for each of the locations on the spatial domain of data considered, supporting intensities are these, which are solely based on diffuse reflectance phenomena and with use of distinct directional parameters of incidence light, in relation to the data substituted.

Hence, it has been postulated in the works, related in this paper, that some illumination set should be used, with at least three directional light sources. Not involving too much into detailed set of assumption taken in theoretical considerations, the set of six and more (usually eight) directional light sources has been used...
in practice, with some success, in visual inspection of surface of metallic and glossy reflectance character.

Fig. 3. Left: overdetermined and strictly determined the aimed task in PSM modified method, right: 5 points vectors intensities taken minutely into considerations in initial data classification

Rys. 3. Po lewej: nadokreśloność i ścisła określenie zadania w zmodyfikowanej metodzie PSM, po prawej: wektory intensywności 5 punktów wzięte pod uwagę we wstępnej klasyfikacji danych

What is more, taken in theory strong assumption of point nature of light sources and constant directions of incidence light, cast onto surface regarded, are softened in practice, but simultaneously this very practice proves, that the two strong assumptions, relating to geometry of acquisition set, still hold. Usually the diameter of field of active light radiation for used light sources, regardless of their light wave emission mechanism, are at least twenty times less, than distances between points of their placements and the section of the surface regarded. Thus, the assumption of point light sources used, as well as, assumption of constant directions of incidence light cast in surface illumination are both satisfied, with accuracy required in visual inspection and further data processing.
From within Computer Vision methods, possibly adaptable to hard initial conditions of data acquisition and data processing in visual inspection of the surface, the PSM methods have been chosen, as those of the most available potential adaptability to the aimed task. Accordingly to general classification of the mostly met multimage approaches to Computer Vision methods (Fig. 4), PSM fits well to cases with static scenery, however with several light sources of point nature, activated, each of them, singly at one data acquisition step over the surface regarded. Other approaches are rather hard or even impossible to adapt, in task of visual inspection of very textured surfaces. Moreover, light sources, characterised by spatial extension, as well as, simultaneously activated, would force the use of the most evolved methods, which lead to absolute solutions, such as SVD decomposition. Yet, the scope of this paper is to give robust solution to problem of visual inspection of surface of stalled object accordingly to observation points, however with its complex topographic and reflectance borne features.

The use of PSM in classical form require use of three directional light sources, activated singly and subsequently over the surface examined with simultaneously realised one at a time single acquisition step of 2D intensity images. That means, that for PSM in original form, one deals with dataset of three 2D intensity images. However in hard initial conditions of the acquisition step, one would rather to use more, than assumed in theory, three directional light sources. In practice, the more light sources are used at acquisition stage, with more 2D intensity images.
collected, the less is the risk of obtaining under-determined task. The task, which consists of acquiring light reflectance coefficient (that is scalar quantity) and moreover: \( N \) vector normal to the surface regarded (rendered in further data processing stage as gradient field contents):

\[
\rho(x, y) = \left\| L^T \cdot L \right\|^{-1} \cdot \left\| L^T \cdot I \right\| \text{ at } \left\| N \right\| = 1
\]  

(1)

\[
\rho(x, y) = \left\| L^T \cdot I \right\| \text{ at } \left\| N \right\| = 1
\]  

(2)

\[
\frac{N_{\text{xy}}}{\sqrt{1 + p^2 + q^2}} = \frac{\left(L^T \cdot I_{\text{ax}}\right)^{-1} \cdot L^T \cdot I_{\text{ax}}}{\rho(x, y)}
\]  

(3)

with \( i \notin \{\text{specularities, self-masking, self-shadowing}\} \)

\[
\frac{N_{\text{xy}}}{\sqrt{1 + p^2 + q^2}} = \frac{L^T \cdot I_{\text{ax}}}{\rho(x, y)} \text{, with } i \notin \{\text{spec, self-msk, self-shdw}\}
\]  

(4)

In relationships (1) and (3) a generalized Moore-Penrose pseudo-inversion of \( L \) matrix of directions \([4, 6]\) for all light sources, used in activation within illumination set, has been adopted, in determination of reflectance coefficient, and of \( N \) vector normal to the surface regarded, respectively. An innovative, in Computer Vision literature, approach has been also included, this based on Singular Value Decomposition \([2, 4]\) in obtaining a pseudo-inverted \( L \) matrix:

\[
\begin{bmatrix} U \end{bmatrix}_{n \times 3} \cdot \begin{bmatrix} S \end{bmatrix}_{3 \times 3} \cdot \begin{bmatrix} V \end{bmatrix}_{3 \times 3} = \text{SVD-decomp}(L)_{n \times 3}
\]  

(5)

In relationship (5) above, \( S \) matrix of SVD is diagonal, hence:

\[
\begin{bmatrix} S \end{bmatrix}_{3 \times 3} = \begin{bmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_3 \end{bmatrix}, \Rightarrow \begin{bmatrix} S^{-1} \end{bmatrix}_{3 \times 3} = \left[ \begin{array}{ccc} \frac{1}{\delta_1} & 0 & 0 \\ 0 & \frac{1}{\delta_2} & 0 \\ 0 & 0 & \frac{1}{\delta_3} \end{array} \right]
\]  

(6)

Resuming, \( L \) matrix can be actually rectangular, rendering the aimed task of determination of desired quantities more flexible and robust in presence of complex topography- and reflectance-borne examined surface features.

In two schemes placed in this chapter, some results are given and presented by illustrative means. In Fig. 5 above, the use of PSM *without* double thresholding, in possible classification and invalid data exclusion has been presented, while in Fig. 6 below, the results with use of initial data classification and their possible exclusion have been presented. In second case, the double thresholding,
embedded within whole data processing framework, results in avoiding apparent distortions of the semi-sphere, which did occur in previous case (Fig. 5).

Fig. 5. Exemplary use of PSM in 3D reconstruction without intensity thresholding

Rys. 5. Przykładowe wykorzystanie PSM w rekonstrukcji 3D bez progowania intensywności

However, in spite of obvious need of such data initially realised treatment, there is also occurrence of unique PSM feature, which in peculiar manner, positively distinct these approaches from others. Namely, in case of lack of stage of initial data classification, in eventually realised their exclusion, there does occur some kind of directional, weighted averaging and autobalancing of the results. In data processing both: intensities which are not originated from strictly diffuse light reflectance phenomena, and intensities which are originated from strictly diffuse reflections, are weighted together directionally and represented in output results.
4. VECTOR GRADIENT FIELD NUMERICAL INTEGRATION METHODS

At first glance, in attempt to chose robust and efficiently implemented method in numerical integration of gradient field (based solely on Computer Vision literature) one could come to elusive conclusion, that variationally and spatially realised approach to the problem was the only valid one. Unfortunately, these approaches have been generally based in their consideration on single-image approach, such as SFS [9, 10, 12, 13].

In case of vector gradient field contents, obtained due to use of first and second stage in data processing framework in PSM, usually, there is small data inconsistency within them, to be neglected in considerations. In some peculiar cases, the level of gradient field distortions are minute or at acceptable level. In consequence, the resulted 3D reconstructed surface, usually is characterised with high level of fidelity in mapping of the topographic features on it, or with slightly occurred reference plane inclination.

Peculiar and advantageous feature of PSM methods, which distinctly differ them from other Computer Vision approaches is the following: with more directional light sources used in data acquisition stage, the resulted vector gradient field
inconsistency is diminished gradually [11]. However, one cannot totally ignore the presence of noises within dataset of acquired 2D image intensities and consequently within determined gradient field.

In pursuit of the solution in the problem discussed and rendered in such a manner, the authors tried with some success the discrete finite Taylor expansion of the surface reconstructed in function of known slopes of it on X, Y directions respectively, i.e in \( p, q \) function [2, 3]. Here, only the basic and some of the most relevant mathematical implication will be given:

\[
 z_{i,j} = z_{i-1,j-1} + \varepsilon \cdot p_{i-1,j-1} + \varepsilon \cdot q_{i-1,j-1} + \varepsilon^2 (p_{i,j-1} - p_{i-1,j-1} + q_{i-1,j} - q_{i-1,j-1}) / 2
\]

\[ z_{i,j}^{n+1} = z_{i-1,j-1}^{n+1} + \varepsilon \cdot p_{i-1,j-1} + \varepsilon \cdot q_{i-1,j-1} + \varepsilon^2 (p_{i,j-1} - p_{i-1,j-1} + q_{i-1,j} - q_{i-1,j-1}) / 2 \]

In relationship (7) both non-iterative and iterative formula for determination of unknown \( z \) function have been given. Further, such a point-wisely realised scheme of numerical integration with skewly oriented path of integration can be substituted with success with three mutually crossed directions, at each point on discrete lattice of the data integrated:

\[
 z_{i,j}^n = \frac{1}{3} (z_{i-1,j-1}^{n-1} + z_{i-1,j}^{n-1} + z_{i,j-1}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j-1} + \varepsilon^2 / 3 (p_{i,j-1} - p_{i-1,j-1} + q_{i-1,j} - q_{i-1,j-1}) \]

Consequently, below in this paper, the so called a triplet of the three integration paths, still holds in pointwise integration scheme. In more evolved scheme, called here below a mask integration scheme, jointly more than one points of the data integrated will be considered, giving in addition an integration formula, in distant analogy to iterative Jacobi’s solutions:

\[
 z_{i,j}^n = \frac{1}{3} (z_{i-1,j-1}^{n-1} + z_{i-1,j}^{n-1} + z_{i,j-1}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j-1} + \frac{2}{3} \varepsilon^2 q_{i-1,j-1} + \varepsilon^2 / 3 (p_{i,j-1} - p_{i-1,j-1} + q_{i-1,j} - q_{i-1,j-1}) \]

\[ z_{i,j}^{n+1} = \frac{1}{3} (z_{i-1,j-1}^{n+1} + z_{i-1,j}^{n+1} + z_{i,j-1}^{n+1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j-1} + \frac{2}{3} \varepsilon^2 q_{i-1,j-1} + \varepsilon^2 / 3 (p_{i,j-1} - p_{i-1,j-1} + q_{i-1,j} - q_{i-1,j-1}) \]

\[ (9a) \]
\[ z_{i,j}^n = \frac{1}{3} (2z_{i,j-1}^{n-1} + z_{i,j+1}^{n-1} + z_{i,j}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j} + \frac{2}{3} \varepsilon \cdot q_{i-1,j} + \] 
\[ + \frac{\varepsilon^2}{3} (p_{i,j} - p_{i-1,j} + q_{i-1,j+1} - q_{i,j}) \] 
\[ z_{i+1,j}^n = \frac{1}{3} (z_{i,j}^{n-1} + 2z_{i,j+1}^{n-1} + z_{i+1,j}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i,j-1} + \frac{2}{3} \varepsilon \cdot q_{i,j-1} + \] 
\[ + \frac{\varepsilon^2}{3} (p_{i+1,j-1} - p_{i,j-1} + q_{i,j} - q_{i,j-1}) \] 

(9b)

Finally, in some trials and experiments, in both boosting convergence of the iterative algorithm to right solutions, and diminishing computational efforts spent on it, the authors decided to use some distant analogy to Gauss-Seidel iterative solutions, with substantially occurred results:

\[ z_{i,j}^n = \frac{1}{3} (z_{i-1,j-1}^{n-1} + z_{i-1,j}^{n-1} + z_{i,j}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j-1} + \frac{2}{3} \varepsilon \cdot q_{i-1,j-1} + \] 
\[ + \frac{\varepsilon^2}{3} (p_{i,j} - p_{i-1,j} + q_{i-1,j} - q_{i,j}) \] 
\[ z_{i,j+1}^n = \frac{1}{3} (z_{i-1,j}^{n-1} + z_{i,j+1}^{n-1} + z_{i,j}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i-1,j} + \frac{2}{3} \varepsilon \cdot q_{i-1,j} + \] 
\[ + \frac{\varepsilon^2}{3} (p_{i,j} - p_{i-1,j} + q_{i,j} - q_{i-1,j}) \] 
\[ z_{i+1,j}^n = \frac{1}{3} (z_{i,j+1}^{n-1} + z_{i+1,j}^{n-1} + z_{i,j}^{n-1}) + \frac{2}{3} \varepsilon \cdot p_{i+1,j-1} + \frac{2}{3} \varepsilon \cdot q_{i+1,j-1} + \] 
\[ + \frac{\varepsilon^2}{3} (p_{i+1,j} - p_{i+1,j-1} + q_{i+1,j} - q_{i,j}) \] 

(10)
Tackling the problems of determination and numerical integration ...

However, still the computational effort spent on it, and convergence in dependence of number of iterations are questionable, in comparison to non-iterative implementation of numerical integration. Moreover, some surface samples, of salient macro-dimensional character in 3D reconstruction process, are biased with locally occurred surface inflation in right bottom corner. In managing these two disadvantages, firstly it was decided to introduce two mutually alternative orientations of the whole data integration paths (called triplets), alongside the same directions, (figures below):

Secondly, it was decided to use both dense and sparse data indexing within lattice of discrete vector gradient field contents, obtaining some kind of slightly crust features on the surfaces reconstructed, in the last case:
5. A POSTULATED CONSISTENT GRADIENT FIELD DETERMINATION PRIOR TO ITS NUMERICAL INTEGRATION

In managing efficiently the whole process of data acquisition and processing in consolidated and automated visual inspection of the very textured surfaces, some postulated methods and implicated constrains have been also introduced, in elaborated trials and experiments. However, so far, at this stage of consideration, these are the only theoretical backgrounds, to be yet implemented and rendered in iterative form.

Not involving into more detailed, thus complicated considerations, here is some relevant proposition for consistent gradient field determination, for variety of the surfaces visually inspected, prior to any attempt of numerical integration, so much elaborated and discussed in the former chapter.

\[
[N_1, N_2, \ldots, N_n]_{3 \times 1}^{(k+1)} = \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times d} \cdot \left[ \begin{array}{c} \mathbf{I} \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{L} \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{N}_1, \mathbf{N}_2, \ldots, \mathbf{N}_n \end{array} \right]_{3 \times 1}^{(k)} + \\
+ \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times d} \cdot \left[ \begin{array}{c} \mathbf{I} \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}_A \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}_B \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}_C \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}_D \end{array} \right]_{3 \times 3} + \\
+ \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times d} \cdot \left[ \begin{array}{c} \mathbf{I} \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{I}^T \end{array} \right]_{3 \times 3} \cdot \left[ \begin{array}{c} \mathbf{C}_1 \end{array} \right]_{3 \times 3} + \left[ \begin{array}{c} \mathbf{C}_2 \end{array} \right]_{3 \times 3} \right]_{3 \times 1}^{(k+1)}
\]
In relations (11) some Richardson simple integration scheme for jointly real-
ised iterative determination of bunch of \( n \) mutually neighbooring \( N \) vectors has been presented. Moreover, this stage of iterative gradient field determinations, which can follow with success the stage described in relationship (1-4), has been additionally supplied with \( C_1 \) and \( C_2 \) constraints matrixes, with strongly imposed requirements on gradient field consistency:

\[
C_1 = \begin{bmatrix}
    p_A & 0 & 0 & -p_D & 0 & 0 & 0 & 0 & 0 \\
    0 & p_B & 0 & 0 & -p_E & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & p_D & 0 & 0 & -p_G & 0 & 0 \\
    0 & 0 & 0 & 0 & p_E & 0 & 0 & p_H & 0 \\
    p_A & p_B & 0 & -p_D & 0 & 0 & -p_G & 0 & 0 \\
    p_A & 0 & 0 & 0 & 0 & 0 & -p_G & 0 & 0 
\end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
    -q_A & q_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -q_B & q_C & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & q_C & q_D & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & q_D & q_E & 0 & 0 & 0 & 0 \\
    -q_A & 0 & q_C & 0 & 0 & 0 & 0 & 0 & 0 \\
    -q_A & q_B & 0 & -q_D & q_E & 0 & 0 & 0 & 0 
\end{bmatrix}
\]
Here, in relationship (11) \([I]\) or \([II]\) matrices are some rectangular extensions of unit matrix of 3\(\times\)3 elements, in form of stacked or jointed, column-wisely or row-wisely, of the required numbers of 3-elements rows or columns, respectively, adequately to the needs of numerical algorithm implementation.

Hence, non-zero elements, placed on main diagonal of the unit matrix, actually in case of \([II]\)^T or \([II]\) matrix, are placed on tooth-like-shaped path, among other zeroed elements, of horizontally or vertically oriented rectangular matrix.

The relationship (12) is a more evolved formula, as the main aim was to produce a single iterative formula for both: iterative determination/updating contents of \(N\) vectors, and iteratively implemented checking/correcting of gradient fields contents consistency. That means, some of the components of \(N\) vector iteratively determined are rendered in the form of contents of gradient field.

Perhaps, more illustrative would be to give originally defined two separate formula, the first for iterative determination/updating of \(N\) vectors:

\[
[N, N_2, \ldots, N_n]^{(k+1)}_{3\times n} = \left( \begin{bmatrix} \hat{I}^T & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right) \cdot \left( \begin{bmatrix} \hat{I}^T & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right)^{-1} \cdot \left( \begin{bmatrix} \hat{I}^T & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right) \cdot \left( \begin{bmatrix} \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right) \cdot [L]_{6\times 3} \cdot [N, N_2, \ldots, N_n]^{(k)}_{3\times n} + \\
\left( \begin{bmatrix} \hat{I}^T & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right)^{-1} \cdot \left( \begin{bmatrix} \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right) \cdot \left( \begin{bmatrix} \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \\ \hat{I} & \hat{I} & \hat{I} \end{bmatrix} \right) \cdot \left( \begin{bmatrix} I_A & I_B & I_C \\ \rho_A & \rho_B & \rho_C \end{bmatrix} \right) \cdot \left( \begin{bmatrix} I_D \rho_D \end{bmatrix} \right)_{4\times 1} \right)
\]

and the second for checking/correcting vector gradient field consistency:

\[
[p]^{(k+1)}_{6\times 1} = ([I]_{6\times 6} - [A_1]_{6\times 6}) \ast [p]^{(k)}_{6\times 1} - [A_2]_{6\times 6} \ast [q]^{(k)}_{6\times 1} \\
[q]^{(k+1)}_{6\times 1} = ([I]_{6\times 6} - [A_2]_{6\times 6}) \ast [q]^{(k)}_{6\times 1} - [A_1]_{6\times 6} \ast [p]^{(k)}_{6\times 1} \tag{14}
\]

where \([p]\) and \([q]\) vectors, and the \(A_1\) and \(A_2\) matrices are jointly, in their contents and meaning, defined quite analogously to \(C_1\) and \(C_2\) matrices.

\[
A_1 \ast [p_A]_{6\times 1} + A_2 \ast [q_A]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_B]_{6\times 1} + A_2 \ast [q_B]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_C]_{6\times 1} + A_2 \ast [q_C]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_D]_{6\times 1} + A_2 \ast [q_D]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_E]_{6\times 1} + A_2 \ast [q_E]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_F]_{6\times 1} + A_2 \ast [q_F]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_G]_{6\times 1} + A_2 \ast [q_G]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]

\[
A_1 \ast [p_H]_{6\times 1} + A_2 \ast [q_H]_{6\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times 1}
\]
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\[ A_1 \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, A_2 \Rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & -1 & 1 & 0 \end{bmatrix} \] (15b)

6. FINALS REMARKS, CONCLUDING REMARKS

In this paper the extensive works have been related on theoretical background, provided with some surfaces in their visual inspections, in presence of hard initial data acquisition conditions. Though, there do really exist some relevant results, in form of comparative consistent statistical analysis [16-20], the aim and rationale of the paper was to present in illustrative manner, the whole methodology in PSM use.

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OMÓWIENIE ZAGADNIEŃ WYZNACZANIA
I NUMERYCZNEGO CAŁKOWANIA
ZAWARTOŚCI WEKTOROWYCH PÓŁ GRADIENTU
W MOŻLIWEJ INSPEKCJI WIZUALNEJ POWIERZCHNI
O ROZBUDOWANEJ STRUKTURZE GEOMETRYCZNEJ

Streszczenie

W artykule przedstawiono informacje dotyczące niezawodnego pozyskiwania i numerycznego całkowania zawartości wektorowego pola gradientu w obecności szumu względem sygnału informacji użytecznej. W zagadnieniu inspekcji wizualnej powierzchni o rozbudowanej strukturze wykorzystano wieloobrazowe podejście, jako nową koncepcję w bezdotykowym badaniu powierzchni charakteryzujących się zarówno złożonością cech natury topograficznej, jak i uwarunkowanych odbiciem światła.

Słowa kluczowe: inspekcja wizualna, Photometric Stereo, bezdotykowe badanie powierzchni